**MINIMUM VALUE COMPUTATION**

Over all real numbers, find the minimum value of a positive real number, y such that

At the minimum value of y, the rate of change in x will equal zero, and the second rate of change in x will result in a positive value, indicating that the curve changes direction at that point from sloping downwards to sloping upwards.

Hence,

Thus, to compute the first order differential, split the equation into two such that

Therefore,

Since both a and b are composite, it can be differentiated using chain rule thus,

Bearing the general form of the differential of a square root, which results in a negative power of half and a scalar multiplication by half, in mind, this results in

In the same way, differentiation the function, b, results in

Therefore, substituting back for the full differential in y gives

At any turning point (maximum or minimum) of the function, this differential equals zero. Therefore, equating to zero and solving for the value of x would result in

Multiplying through by the LCM of the denominators being to remove the denominators results in

Rearranging by moving one term to the left hand side and squaring both sides to eliminate the square roots gives

Expanding hence

Subtracting the first term from the both sides of the equation results in

which on expanding using the appropriate Pascal’s coefficients gives

Collecting all the terms to the left hand side of the equation yields

Removing common factors by dividing through by 24 gives

Solving this expression using any tool results in the following optimum values of x for which y is either a maximum or minimum

Considering that from the structure of the first differential, it can be implied that computing a second differential might be burdensome, a simple search method can be carried out using these optimum values of x to determine that which gives the minimum value of y. This will be done via substitution into the function, y, thus

When x = ,

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Thus, the x value of gives the smaller value of y from this test, and since this has been proven to be a turning point, the value of y at the point is then the minimum value.

Therefore, the minimum value of y is